

# Downward Real Wage Rigidity and Equal Treatment Wage Contracts: Theory and Evidence\*

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## Abstract

Recent dynamic contracting models of downward real wage rigidity with “equal treatment” — newly hired workers cannot price themselves into jobs by undercutting incumbents — imply that real wages are relatively rigid in “bad” times but upwardly flexible during “good” times. We use an administrative panel dataset to establish that such asymmetries are a feature of West German labor markets. We find that the elasticity of real wages with respect to output is very close to zero in downswings but positive and highly significant in upswings. In a separate analysis we find that after controlling for match fixed effects, the cyclicalities of new hire wages is approximately the same as that for incumbent wages, regardless of whether or not they joined the establishment from unemployment. This is supportive of equal treatment. We also show that a four parameter version of the equal treatment contracting model of Snell and Thomas (2010) can replicate reasonably well the salient time series properties and co-properties of real wages, output, and unemployment, in particular the asymmetric response of wages to output that we find in the data.

**JEL classification:** E24; E32; C23.

**Keywords:** Business Cycles; Employment; Unemployment; Real Wage Rigidity; GDP growth;

Models with Panel Data.

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# 1 Introduction and Overview

There is now a sizeable literature devoted to estimating the cyclicalities of real wages using large panel datasets to correct for composition bias (e.g., Solon et al. (1994) for the US, Devereux and Hart (2006) for the UK, Martins et al. (2012) for Portugal, and Stüber (2017) for Germany). However, recent models of dynamic wage contracting that exhibit downward real wage rigidity (in particular the models of Menzio and Moen (2010) and Snell and Thomas (2010)) suggest that wages will be asymmetrically cyclical — relatively rigid in “downswings” when productivity and output are falling but upwardly flexible during “upswings” when productivity and output are rising.<sup>1</sup> Such asymmetries are important for unemployment (and output) dynamics as we will show. They also matter for the *long-run level* of unemployment; Benigno et al. (2015) argue that long-run unemployment rates and the variance of productivity growth are positively associated. They explain this finding by asymmetric wage adjustments to productivity growth of the type we argue for here.<sup>2</sup>

In this paper we use a sample of workers drawn from an administrative German panel dataset for the years 1978–2014 to establish that — defining up- and downswings in terms of positive and negative growth in real GDP per capita — asymmetries as predicted by models of dynamic wage contracting that exhibit downward real wage rigidity seem to be a feature of West German<sup>3</sup> labor markets. The key empirical feature to emerge from our study is that the elasticity of real wages with respect to GDP per capita (henceforth just “elasticity”<sup>4</sup> and henceforth real GDP per capita referred to as just “output” for brevity) is close to zero and insignificant in downswings. However, in upswings the elasticity is positive and statistically significant. In a separate analysis we find that after controlling for match fixed effects the cyclicalities of new hire wages is not significantly

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<sup>1</sup>The sign correlation of output and productivity growth is close to but not exactly 100% in our theoretical model. There are some rare occasions when productivity suffers a small drop but output rises. In our simulations these events occur less than 2% of the time.

<sup>2</sup>Using an *ad hoc* model of downward rigidity they demonstrate that higher variance of productivity growth translates into higher unemployment as downturns lead to higher unemployment whereas in upturns unemployment is the same as full employment ensues; a similar effect applies if trend productivity growth falls, again leading to higher unemployment.

<sup>3</sup>All of the data in this paper, apart from the total factor productivity (TFP) and the consumer price index (CPI) series, are for West Germany (excluding Berlin) only (the data for TFP and CPI are for West Germany prior to 1991 and the whole of Germany afterwards). Henceforth and unless otherwise stated we refer to West Germany as just “Germany” for brevity.

<sup>4</sup>Use of the term “elasticity” to denote the regression coefficient of wage on output growth is somewhat misleading as our estimate has no clear structural interpretation at this point. We use the term for brevity and for consistency with the cognate empirical literature.

different from that of incumbents, regardless of which cyclical indicator is used (unemployment or output) and regardless of whether or not the worker was employed immediately before joining the firm. This result is particularly important: if wages are downwardly rigid in downswings then it suggests that the unemployed cannot price themselves into jobs by “undercutting” incumbents and hence labor markets will not clear (see, e.g., Hall, 2005; Gertler and Trigari, 2009; Snell and Thomas, 2010).

In the second part of this paper we show that a simple three or four parameter version of Snell and Thomas’s (2010) equal treatment contracting model can generate a strong positive elasticity in upswings and an elasticity close to zero in downswings. In the model, risk neutral firms have an incentive to smooth the wage profiles of risk averse workers by offering a contract that limits the rate at which real wages fall in downswings. The rate of fall is the result of a trade-off for the firm between wanting to avail itself of cheaper new hires on the one hand and not wanting to create too much variability for the incumbents on the other. This results in a maximum rate of wage fall no matter how cheap new hires potentially become. The existence of an equal treatment constraint — new hires must be paid the same as incumbents — implies that the unemployed cannot price themselves into jobs by offering to undercut incumbent wages. As a result, if the economy suffers a negative and persistent productivity shock, downwardly sluggish wages will generate unemployment that will endure until either productivity recovers or until wages have fallen sufficiently far to clear the labor market. In this scenario, real wages will exhibit little covariation with output when productivity is falling. By contrast, wages adjust rapidly (upwards) in good times when productivity is high so that in upswings they will be strongly positively correlated with output. The reason for the asymmetric response is that workers are ex post mobile, so that in good times their outside opportunity has a high value; firms have no option but to raise wages to match this and so wages are upwardly flexible during upswings.

The model requires total factor productivity (TFP) as a sole input. To assess the model’s fit to the data we follow the real business cycle (RBC) literature and generate a synthetic TFP series whose growth and first order autocorrelation matches that of German TFP growth. Using this series, the model generates wage and unemployment series that display the same asymmetric response patterns we found in the data. In addition the first order autocorrelation coefficients and variances of the model’s simulated real wage growth, unemployment and output growth broadly

match their counterparts in the data. As far as we know, this is the first paper not only to look at the empirical downward real rigidity of wages for workers including new hires, but also to simulate a microfounded model that is capable of matching these observed regularities.<sup>5</sup>

The outline of the paper is as follows. In Section 2 we review both the empirical and theoretical literature on downward rigidity. In Section 3 we present an overview of the data. In section 4 we present tests of equal treatment and the evidence for asymmetric wage cyclicality. In section 5 we present two versions of the model and simulate them to see if predictions for output, unemployment and wages match those of the data. In Section 6 we consider robustness checks and other caveats: the main results on asymmetry are robust with respect to dropping the “German reunification years” from the sample and dropping the early years from the sample. The results on equal treatment — that new hires have the same wage cyclicality as incumbents — are robust with respect to increasing the length of time an employee remains a new hire after joining the firm. We also show that the asymmetry result holds when we split the data into six broad industrial sectors providing some reassurance that the asymmetry is a pervasive feature of the West German economy. Section 7 contains concluding comments.

## 2 Related Empirical and Theoretical Literature

### Empirics

Evidence for downward real wage rigidity (DRWR) has come in different forms, although the topic is understudied relative to downward nominal rigidity<sup>6,7</sup>. Broadly, there are three methodologically distinct approaches.

The first and most common approach has been to look for DRWR by comparing empirical wage-change distributions with notional distributions constructed under assumptions about how wage changes would be distributed were there no downward rigidities. The International Wage Flexibility

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<sup>5</sup>Snell and Thomas (2010) only perform unemployment simulations.

<sup>6</sup>For a very useful survey of much of the literature relevant to downward nominal rigidity, and its implications for cyclicality of the relevant real cost of new hires, see Basu and House (2016). Their DSGE analysis, however, suggests that price rigidities play a more important role than nominal wage rigidities in generating fluctuations.

<sup>7</sup>Recent analyses have simultaneously estimated the incidence of both real and nominal downward rigidities. Doing so tends to reduce the estimated incidence of the latter relative to the former, see Goette et al. (2007).

Project which analyzed employee wage data from 16 OECD countries found evidence of wage changes being clumped around expected inflation in number of countries, and under assumptions about the shape of the notional distribution, fewer than the expected number of changes below the expected inflation rate, consistent with a degree of DRWR (for a summary, see Dickens et al. (2007); Dickens and Goette (2005) discuss the methodology in detail). Germany is around average in terms of the degree of wage rigidity by these measures. In a related analysis, Bauer et al. (2007) find substantial DRWR in Germany, with 30–70% of wages being subject to real rigidity. These studies typically use a model in which DRWR occurs as an absolute constraint on some wage changes, whereas we will allow for wage cuts that are dampened relative to wage increases. In addition we will be interested in how wages change over the business cycle, an issue that is not directly addressed in this literature which looks for incidence of downward-constrained individual wage changes.<sup>8</sup> Finally, the wage changes studied are for those in continuing employment. We are also interested in the cost of *new hires*, as that is likely to be particularly relevant for hiring decisions.<sup>9</sup>

A second source of evidence comes from surveys and interviews. Bewley (1999, pp. 208–209) finds for the US considerable evidence from unstructured interviews of both nominal and real wage downward rigidity, although for the latter the degree is not absolute and some firms were prepared to let real wages fall somewhat. Similar findings exist in a number of more structured surveys. Most relevant to the German labor market, Franz and Pfeiffer (2006), based on a survey of German firms and largely following the methodology of Campbell and Kamlani (1997), found that rigidity was accounted partly for by a desire to avoid fluctuations in wages in line with wage smoothing (less so amongst the highly skilled), although union agreements were cited as an important factor,<sup>10</sup>

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<sup>8</sup>More precisely, the aim is to estimate the fraction of workers who, if they were scheduled for a real wage cut, actually receive one. This is independent of the business cycle: in a boom we would expect fewer workers to be scheduled for a cut but the idea is that the fraction of this smaller number actually receiving a cut is similar to the corresponding fraction in a recession.

<sup>9</sup>Holden and Wulfsberg (2009) do partially address the issue. They argue that firms may sidestep downward rigidities for example by cutting wages for new hires, or jobs may be shifted to firms with lower wages. To address this, and in a reversal of the usual composition arguments following Solon et al. (1994) discussed below, they look at *industry level average* wage changes across 19 OECD countries for the period 1973–1999, arguing that the average wage will capture the overall average cost of labor. Notional wage change distributions are estimated from years with high nominal and real wage growth, the idea being that downward rigidities would not come into play in such years, and actual distributions are compared. They find some evidence of DRWR in a subset of countries.

<sup>10</sup>Nevertheless, of the responding firms only 38% apply union contracts; even amongst these around half were paying “effective” wages above the collectively bargained wage, and overall there is some individual firm-worker bargaining over wages, either independently of or in addition to union contracts, in 83% of the firms. Babecky et al. (2010), in a cross-country study (but excluding Germany), find a positive relationship between collective bargaining coverage

and a number of statements in line with efficiency wage theory also received assent.

A third approach is to take a more standard microeconomic model which models the real wage of each worker in terms of cyclical variables such as unemployment (common to all workers) as well as other control variables. In this paper we adopt the methodology of the microeconomic wage cyclicality literature, following Solon et al. (1994), in which composition bias — the tendency for lower paid workers to differentially end up unemployed in downturns, thus biasing downwards estimates of cyclicality based on average wages — is corrected for in a first stage regression that extracts a series of time effects. In a second stage we look to see if this series exhibits some form of downward rigidity with respect to cyclical variables. Martins (2007) has performed an exercise along these lines for Portugal. He defines upswings (downswings) in terms of rising (falling) unemployment rather than output growth. Using this definition he examines the extent of differential real wage cyclicality in the two regimes. However he finds that the semi elasticity of wages with respect to unemployment actually *increases* (in absolute value) when unemployment is rising.<sup>11</sup> The result that wages are relatively inflexible in times of falling unemployment but highly flexible when it is rising is hard to rationalize. Shin and Shin (2008) also estimate asymmetric wage responses to unemployment. However they focus on asymmetries across workers of different tenure at a point in time rather than asymmetries across different time periods for all workers. The asymmetries they find are consistent with what one would find in a Beaudry and DiNardo (1991) world of implicit contracts under worker mobility.<sup>12</sup> By contrast, when we measure cyclicality in terms of rising and falling output, we find evidence that all workers *regardless of tenure* are subject to the same wage cyclicality and that the effects are asymmetric across time rather than across tenure. In particular, when we estimate the composition and match quality controlled average wages of a) new hires from employment and b) new hires from unemployment they appear to have the same (asymmetric) cyclicality to other workers (incumbents). In addition Gartner et al. (2013) look at asymmetric wage adjustment with respect to unemployment changes for Germany, 1995–2004, for small and medium sized firms (fewer than 500 employees). They find evidence that wages adjust less in downswings to changes in regional unemployment, and that uncovered wages are more flexible

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and downward real wage rigidity, where the latter is defined as some form of wage indexation.

<sup>11</sup>In addition he looks at each worker’s wage growth rather than its level — a classic method to remove various fixed effects but also a limitation as it means that new hires must be omitted from the sample.

<sup>12</sup>In effect they find in favor of “minu” — the minimum unemployment rate during a worker’s tenure — which Beaudry and DiNardo (1991) argue determines wages in this scenario and which has found to be significant in a host of other papers.

than covered union wages in upswings, but uncovered wages do not adjust to rising unemployment.

None of the existing studies, as far as we are aware, have analyzed downward rigidity of wages of new hires, something integral to our analysis. Certainly existing wage cyclicality studies, following Bils (1985), suggest that wages of job “movers” are considerably more cyclical than “stayers”, while Haefke et al. (2013) establish comparable results in the CPS for workers moving from non-employment to employment. Likewise Farès and Lemieux (2001) argue that in Canada new entrants bear most of the adjustments in real wages over the business cycle. These results then suggest wages of new hires have different cyclical properties to those in ongoing employment, so that any DRWR identified for those in continuing employment might not carry over to new hires. Likewise other papers have found evidence to support the existence of bilateral worker-firm implicit wage contracts (so that new hire wage contracts are bargained independently of existing contracts within a firm, with the implication that the cyclical properties of the start wage will be different from that of ongoing wages within the match); see for example Devereux and Hart (2007), Grant (2003), and the original seminal paper by Beaudry and DiNardo (1991). Both of these conclusions are controversial, however. Gertler and Trigari (2009) dispute whether new hire wages are in fact more cyclical: they suggest that cyclical upgrading may contaminate these new hire estimates, if say skilled workers end up taking less skilled jobs in a recession; the lower wage of such a “mover” in a recession might not be due to wage cuts within the less skilled jobs. In their study on US data controlling for job match effects offers support for equal treatment. Similarly, Hagedorn and Manovskii (2013) show that failing to control rigorously for worker-firm match (and within firm-job match) quality can lead to spurious tenure specific cyclicality of the kind consistent with implicit contracts and excess new-hire wage cyclicality. Following these arguments about the importance of controlling properly for match quality we include match fixed effects as in Gertler and Trigari (2009). Doing so we find no differential cyclicality across workers (new hires and incumbents). Instead we find differential cyclicality across time (upswings and downswings). In short we find that equal treatment and asymmetric adjustment are a key feature of German wages. In a further analysis we find that the magnitudes of the effects are broadly in line with those elicited from a calibrated version of Snell and Thomas’s (2010) model of wage contracting subject to equal treatment. We now discuss this theory in the context of other models of real wage rigidity.

## Theory

There are very few theories that result in downward (but not upward) rigidity.<sup>13</sup> Most efficiency wage models imply real wages that are too high for market clearing and are rigid in both directions (see Babecky et al. (2010) for further discussion). Models such as Hall (2005), Thomas and Worrall (1988), Rudanko (2009)<sup>14</sup> or MacLeod and Malcomson (1993) have wages that are both downward *and* upward rigid for a range of shocks, and outside that range adjust to external conditions.<sup>15</sup> Likewise models of worker-firm bargaining where workers are subject to habit formation or loss aversion can exhibit a range of real wage inertia, following Bhaskar (1990), but not readily asymmetry. A recent example is Holden and Wulfsberg (2009).<sup>16</sup> Insider-outsider theories where insiders, who are protected from layoffs, may be able to oppose wage cuts in recessions, but bargain for wage increases in booms, can in principle deliver asymmetric wage responses; see Lindbeck and Snower (2001). We simulate the contracting model of Snell and Thomas (2010) in Section 5 (where it is discussed further) which does have asymmetric responses of wages to up- and downturns, similar to Menzio and Moen (2010). An interesting model which shares some features with the latter two papers, is Bils et al. (2016), where a matching model with staggered wage bargaining is analyzed; effort is endogenous and sticky wages can lead to variation in effort of incumbents which transmits stickiness (but not asymmetry), via bargaining, to new hire wages.

Two other issues have arisen in recent debate relevant to this paper. First, of particular interest from the point of view of allocation, is not the average wage nor even the wage paid to new hires, but what Kudlyak (2014) calls the “user cost of labor”, the cost of hiring a worker today relative to postponing this decision one period, which takes into account future wages and is the appropriate allocational variable (see also Basu and House (2016)). Assuming stickiness of real wages of those in continuing employment, she shows that the user cost may be much more cyclical than the new

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<sup>13</sup>See Abbritti and Fahr (2013) for a recent example where asymmetric nominal wage adjustment costs are *assumed*, and embedded in a New Keynesian model with search frictions.

<sup>14</sup>Rudanko (2009), who incorporates the limited-commitment Thomas-Worrall model into a directed search framework, shows that despite the within-match wage stickiness, this only has a marginal effect on job creation as any stickiness does not substantially transmit to the overall discounted cost of a new hire.

<sup>15</sup>In the latter model, with general investments and if there is inflation, nominal wages might be either rigid or move upwards, with rare downwards movements, because the worker’s outside option will typically be rising in nominal terms. With contracts specifying real wages there is no presumption of asymmetry.

<sup>16</sup>They show that loss aversion can lead workers to bargain harder if their wage would fall leading to a higher real wage in a downturn relative to what they would get in the absence of loss aversion. However this does not lead to an asymmetric relationship between wages and outside options.

hire wage as, say, a low new hire wage in a recession will be followed by lower than average wages for that worker in the future. This implies a new hire at a low wage also tends to involve future wage savings relative to later hires made when the market has recovered, so that the user cost actually falls proportionately more than the wage in recessions, and vice versa in booms. The model we simulate below is one of equal treatment — workers that differ solely according to when they are hired by a firm will be paid the same within that firm. Because of equal treatment the future wage savings in a recession are by definition zero, and the user cost coincides with the new hire wage, which is therefore allocational. A second, related, issue arises with downward rigidities, namely that if they are anticipated, a firm will want to hold down current wages to relax future downward constraints (Elsby (2009)). Because downward rigidity in the model is a feature of optimal contracts rather than a constraint, the solution already takes this into account.

### 3 Data

Our panel of workers is drawn from the IAB Beschäftigten-Historik (BeH), the Employee History File of the Institute for Employment Research (IAB) of the German Federal Employment Agency. The BeH consists of data on the total of gainfully employed members of the German population who are covered by the social security system.<sup>17</sup> Not covered are self-employed, family workers assisting in the operation of a family business, civil servants (Beamten), and regular students. The BeH covers roughly 80% of the German workforce. Plausibility checks performed by the social security institutions and the existence of legal sanctions for misreporting guarantee that the earnings data are very reliable.

Due to protection of data privacy, we are not allowed to work with the entirety of the BeH. From the BeH (version 10.01) we use a 20% random sample of all workers that worked full-time<sup>18</sup> in at least one year since 1975. The BeH is organized by employment spells. A *spell* is a continuous period of employment within a job within an establishment in a particular calendar year. Hence

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<sup>17</sup>The BeH also comprises marginal part-time workers employed since 1999.

<sup>18</sup>More precisely we focus on “regular workers” according to the definition used in the Administrative Wage and Labor Market Flow Panel (AWFP) dataset. In the AWFP a person is defined as a “regular worker” when she is full-time employed and belongs to person group 101 (employee s.t. social security without special features), 140 (seamen) or 143 (maritime pilots) in the BeH (see Seth and Stüber, 2017; Stüber and Seth, 2017). Therefore all (marginal) part-time employees, employees in partial retirement, interns etc. are not accounted for as regular workers.

the maximum spell length is 366 days. For each identified full-time worker, the BeH has a record of all existing employment spells — including part-time employment, apprenticeships etc. For our analyses we restrict attention to employment spells of full-time workers<sup>19</sup> aged 16 to 65 years from West Germany and the years 1978 to 2014. We only keep employment spells if the workers are employed on December 31<sup>st</sup> of the respective year.<sup>20</sup> We define a newly hired spell as the first spell of a worker at the firm.<sup>21</sup> Hence, the tenure of a worker in an establishment which spans more than one calendar year will consist of multiple spells, with the first being classified as a new hire spell. We also calculate tenure from the dataset. All data are collected at the establishment level but henceforth we refer to establishments as “firms” in keeping with the phrasing used in the discussion of the theory.<sup>22</sup>

Our dependent variable is the real average daily wage of worker over any spell. As the earnings data are right-censored at the contribution assessment ceiling (“Beitragsbemessungsgrenze”), only non-censored wage spells are considered in the analyses.<sup>23</sup> To calculate the average daily real wage and real output per capita in 2010 prices we use the German Consumer Price Index (CPI, see Table A2).

Further we drop all spells that have missing tenure. This means a worker only enters the data

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<sup>19</sup>The BeH only documents total spell earnings and not hours worked in that spell. We therefore only consider full-time workers as these workers’ hours are likely to be acyclical. In Section 6 we analyze the time series properties of an extraneous estimate of the average hours worked in a year by full-time employees in Germany. We find cyclicity — in the sense of having a significant correlation with output — to be relatively weak.

<sup>20</sup>This implies we only ever have a maximum of one spell per worker per year so that when we compute yearly averages over spells we do not weight more heavily those workers with multiple within year spells. It also excludes most short-lived spells in the data in particular temporary summer work.

<sup>21</sup>Re-hires are therefore not identified as new hires. Our decision to treat returning workers as incumbents is because of the relatively short time of absence; 70% of returners were returning after an absence of less than one year and the average length of time away for returners is about 20 months. This suggests that these spells are for workers who have long-term relationships with the firm and whose absences were temporary (for reasons such as paternity/maternity leave). As a final check we repeated our analyses on a dataset that excluded all “returning” spells — all spells where the worker had been away the previous year. There is little quantitative and no qualitative change in either the equal treatment or the asymmetric adjustment results.

<sup>22</sup>The main results of this paper hinge on estimates that control for match fixed effects with the underlying assumption being that matches are with establishments not firms. However, even if matches are formed at the firm level, then using worker-establishment fixed effects will in any event absorb them; their use in this case may be inefficient but would not bias the estimated year effects.

<sup>23</sup>We drop spells with wages  $\geq 0.98$  \* the contribution assessment ceiling. Dropping top-coded spells leads to an under-representation of highly qualified workers, making the results somewhat less generalizable. Because the wages of highly qualified workers are less likely to be covered by a collective bargaining agreement (see, e.g., Düll, 2013) and because uncovered wages are more flexible than covered wages (see, e.g., Devereux and Hart, 2006), it is probable that we slightly underestimate the wage cyclicity. For a quantitative evaluation of the effect of dropping censored spells, see, for example, Appendix A of Stüber and Beissinger (2012).

when she joins a firm after January 1<sup>st</sup>, 1975.<sup>24</sup> This implies we observe relatively few incumbents (relative to new hires that is) in the early years (see Table A1). Our estimation of new hire and incumbent cyclicalities should not be biased by including these early years although the precision of the estimated cyclicalities may well be. However, in a sensitivity analysis we show that our results are largely unaffected when we drop the early years of the sample.

Our upswing/downswing variable is GDP per capita for West Germany excluding West Berlin.<sup>25</sup> This data identifies 11 downswing years.<sup>26</sup> GDP data were obtained from the German Federal Statistical Office and the Federal Statistical Offices of the Federal States. The aggregated unemployment rates were obtained from the Federal Unemployment Agency (see Table A2).

As we noted above, the prime reason for our definition of up- and downswings is that wage contracting models of downward real wage rigidity usually rely on an insurance mechanism whereby risk averse workers are protected from large changes in their wages by risk neutral firms (Menzio and Moen, 2010; Menzio and Shi, 2011). When productivity (and hence output) falls in the model we study (Snell and Thomas, 2010), this insurance mechanism restricts wage falls whilst the necessity to retain workers in competitive labor markets forces firms to raise wages when productivity (and output) recovers.

The final dataset used in our analyses contains over 97.8 million employment spells of nearly 9 million workers working for more than 2.8 million firms.

## 4 Estimation

### 4.1 Testing for Equal Treatment

In this section we wish to assess the extent to which new hire wages are more cyclical than those of incumbents. There is already an extensive literature that does this and we briefly review the key methods here. In order to identify the cyclicalities of wages one first of all needs to control for

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<sup>24</sup>For the analyses we only use the years 1978 to 2014, but for the identification of newly hired workers and the calculation of firm tenure we use BeH data from 1975 onwards.

<sup>25</sup>In a robustness check below we use real GDP; the results are presented in Section B in the Appendix, and are virtually unchanged.

<sup>26</sup>Identified downswing years are indicated by bold year numbers in Table A2.

composition bias. Following Solon et al. (1994) this can be achieved by a two-step method applied to panel data. In the first stage, year effects are extracted from the panel using year dummies whilst controlling for worker-firm characteristics. In the second stage the year effects are regressed on a cyclical indicator — traditionally unemployment, occasionally average labor productivity — to estimate the cyclicity of wages. The use of panel data allows one to control for changes in the average worker quality over the business cycle whilst the use of a two-stage method delivers standard errors that are robust to within time period error clustering. The method is easily extended to examine the cyclicity of different classes of worker — in this context (two types of) new hires and incumbents. In this section we use the two-stage method to test for equal treatment, i.e., to examine whether or not new hire wages are more procyclical than those of incumbents. In the next section we use the method to examine our hypothesized asymmetric wage adjustment.

As noted above it is important to control for as much worker-firm heterogeneity as possible and a natural way to do this is to use worker-firm (match) fixed effects (MFE) as well as proxies for returns to tenure and experience. It is widely believed that match quality is procyclical (see the discussion in Gertler et al. (2016)). Our tests for equal treatment examine the relative cyclicity of new hire versus incumbent wages — so failing to control for procyclical match quality will lead to misleading inferences in this respect (Gertler and Trigari (2009)).

As already stated, we identify new hires by their first spell in a firm. All other workers are “incumbents”. To test for equal treatment we wish to identify three sets of composition-controlled yearly wage averages (henceforth “year effects”): one set for incumbents, one set for new hires joining the firm from unemployment (defined as those not employed<sup>27</sup> for at least 4 weeks prior to joining and referred to as ue’s) and one for those coming from other employment (referred to as ee’s). Our strategy is to estimate the cyclicity of these three sets of year effects and establish whether or not new hire year effects (either ee’s or ue’s) display significantly more cyclicity than those of incumbents. We emphasize that the first stage regressions have no causal or economic interpretation — they are devices to obtain yearly averages of (log) wages whilst controlling for different worker and worker-firm characteristics.

In the first stage the primary specification to be estimated is the panel regression

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<sup>27</sup>Our dataset does not allow us to distinguish between new hires who were previously in the labor force but unemployed from those who were previously not in the labor force.

$$w_{ijt} = m_{ij} + \sum_{\tau=1}^T \beta_{\tau}^I I_{\tau}^t + \sum_{\tau=1}^T \beta_{\tau}^E E_{\tau}^t + \sum_{\tau=1}^T \beta_{\tau}^U U_{\tau}^t + \sum_{k=1}^2 \lambda_k age_{it}^k + \sum_{k=1}^4 \phi_k ten_{ijt}^k + v_{ijt}, \quad (1)$$

where  $w_{ijt}$  is the log of real average daily wage of worker  $i$  in firm  $j$  during year  $t$ , and  $v_{ijt}$  is an error term assumed to be orthogonal to the regressors.

The equation allows for three distinct sets of year effects written in the first three summation terms. The first consists of the dummies  $I_{\tau}^t$  ( $\tau = 1, \dots, 37$ ) with coefficients  $\beta_{\tau}^I$  where  $I_{\tau}^t$  equals one if  $t = \tau$  and the worker is an incumbent but is zero otherwise. The  $\beta^I$  coefficients are the incumbents' year effects. The second (third) set of dummies  $E_{\tau}^t$  ( $U_{\tau}^t$ ) take the value one if the wage is from an ee (ue) new hire and  $t = \tau$  but equal zero otherwise. The  $\beta_{\tau}^E$  ( $\beta_{\tau}^U$ ) are the corresponding year effects. The variable  $age_{it}$  is the worker's age in years and  $ten_{ijt}$  is the worker's firm tenure measured in days at the end of the spell. Finally,  $m_{ij}$  is a MFE. Note that this effect controls for (estimates) the sum of a firm  $j$  effect plus a worker  $i$  effect plus a match quality effect. Whilst it is a general way of absorbing heterogeneity in the panel, a drawback is that if new hire wages are excessively sensitive to the state of the cycle at entry and if (part of) this effect remains constant throughout the entire relationship with the firm then it will be absorbed into the MFE and will not appear as "excess" new hire cyclicalities; for example if new hire effects are procyclical and permanent they will be observationally equivalent to procyclical match quality effects.<sup>28</sup> We return to this issue below where we assess the impact of replacing MFE with less general worker fixed effects (WFE).

To test for equal treatment we see if the wage year effects (henceforth equivalently referred to as "wages") of the three types of workers have differential cyclicalities. In this "second stage" we treat each set of year effects — the  $\beta_{\tau}^k$  coefficients — as separate time series and analyze their respective cyclicalities. Our prime measure of the cycle in this paper is output growth and it is natural we use this to examine the relative cyclicalities of wages of our three different kinds of worker. However the traditional literature on cyclicalities of new hire versus incumbent wages focuses on the

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<sup>28</sup>Consider, for example, a worker who upgrades to a higher wage firm in a boom but who receives in addition a (non-equal treatment) higher wage than equivalent incumbents because of the tight labor market. Suppose first that this higher wage is a temporary hiring premium. Then, as argued by Gertler and Trigari (2009), the MFE will wash out the upgrading and the temporary premium will be picked up in  $\beta_{\tau}^E$ . If this is typical, the  $\beta_{\tau}^E$  series will be more cyclical than  $\beta_{\tau}^I$  and equal treatment is rejected (we find no evidence for such temporary non-equal treatment). Now suppose instead that the premium is a *permanent* addition to the wage; in this case the MFE will absorb this along with the upgrading, and the deviation from equal treatment is not picked up.

aggregate unemployment rate  $u_t$  rather than output. Incentives to deviate from equal treatment are arguably likely to be better related to the rate of unemployment than output; if unemployment is high, workers should be prepared to work at a lower wage than incumbents, while if the labor market is very tight firms may be more prepared to violate internal norms and pay above internal rates.<sup>29</sup> Consequently we conduct a second exercise using this measure also. The two second stage regressions are therefore

$$\Delta\beta_t^k = \alpha + \gamma_{\Delta y}^k \Delta y_t + \varepsilon_t \quad k = I, E, U \quad (2)$$

$$\beta_t^k = \alpha + \gamma^k u_t + \varepsilon_t \quad k = I, E, U \quad (3)$$

where  $y_t$  is the log of output and  $\Delta$  denotes the change in the variable.

The  $\gamma$ 's are not causal parameters. They are merely estimates of the unconditional comovement (i.e., normalized unconditional covariance) between wages (wage growth) and unemployment (output growth). Estimating these unconditional comovements has become the focus of interest in the recent wage cyclicality literature.

One problem with (3) is that both unemployment and wages are highly persistent. Consequently (3) is a specification involving near unit root processes and this raises the possibility of spurious regression in small samples. Interestingly however, the residuals from (3) are less persistent than both wages and unemployment. This suggests that unemployment and wages share a common persistent component.<sup>30</sup>

Turning to (2) this specification is similar to that of Haefke et al. (2013). They use first differences to measure the elasticity of wage growth with respect to the growth of output per worker. The difference here is that we use output per capita. This elasticity —  $\gamma_{\Delta y}^k$  — gives us an alternative and entirely different metric for the cyclicality of wages. Estimates of these quantities

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<sup>29</sup>We are grateful to a referee for stressing this point.

<sup>30</sup>As a further check on our results we also estimated (3) in first differences (to remove the persistence). Whilst this is a different metric of cyclical comovement it does give us a robustness check on the equal treatment results with respect to removing persistence. We find significant coefficients in all cases (albeit estimates that are somewhat smaller in absolute value than their levels counterparts) and support for equal treatment (insignificant extra new hire cyclicality).

for our three classes of worker, incumbents, ee’s and ue’s, are given in Table 1.<sup>31</sup>

Table 1: The Cyclicity of Incumbent and New Hire Wages

Wage Elasticity (Output)	$\gamma_{\Delta y}^I$	$\gamma_{\Delta y}^E$	$\gamma_{\Delta y}^U$	$t_{\gamma_{\Delta y}^E - \gamma_{\Delta y}^I}$	$t_{\gamma_{\Delta y}^U - \gamma_{\Delta y}^I}$
<i>MFE</i>	0.359 (0.089)	0.352 (0.108)	0.419 (0.094)	-0.21	1.56
<i>WFE</i>	0.376 (0.098)	0.567 (0.122)	0.636 (0.156)	3.11	2.55
Wage Semi-Elasticity (Unemployment)	$\gamma^I$	$\gamma^E$	$\gamma^U$	$t_{\gamma^E - \gamma^I}$	$t_{\gamma^U - \gamma^I}$
<i>MFE</i>	-1.293 (0.222)	-1.332 (0.258)	-1.440 (0.243)	-0.52	-1.05
<i>WFE</i>	-1.483 (0.244)	-1.945 (0.303)	-2.23 (0.333)	-2.59	-2.50

Notes: Estimates of the parameters in equations (2) and (3). Robust standard errors using equation (2) and Newey West standard errors with lag order 1 using equation (3) in brackets.  $t_{a-b}$  is the t-test for  $a = b$ . MFE = worker-firm (match) fixed effects, WFE = worker fixed effects, superscript I = incumbent workers, and superscript E (U) = new hires from employment (unemployment).

Table 1 presents estimates using two sets of fixed effects. The first two rows are our main case of match fixed effects as per equation (1). The next two rows replace MFE with WFE.

Starting with the  $\gamma_{\Delta y}^k$  obtained using MFE, we see that there is little difference in incumbents’ wage cyclicity and that of ue’s and ee’s. The t-ratios (columns 5 and 6) confirm that these small differences are wholly insignificant. The semi-elasticities of wages with respect to unemployment are also very close for all three categories of worker and the insignificance of the relevant t-tests confirms this closeness.

It is interesting to compare the WFE estimates with those obtained using MFE. Starting with the former we see these estimates imply substantially higher new hire versus incumbent wage cyclicity. Similar results have been found previously, see Haefke et al. (2013), Stüber (2017), and the literature summary in Pissarides (2009). However when we move to MFE we see that this extra cyclicity disappears. We would argue that this is evidence in favor of procyclical upgrading (where

<sup>31</sup>Standard errors are in brackets: These are robust for the elasticities but for the semi-elasticities where there was substantial error autocorrelation they are Newey West with lag order one. The most persistent of the error terms — from the ue regression — had two significant partial autocorrelations of 0.76 and -0.39 respectively. However Newey standard errors were insensitive to an increase in lag order from one to two so the order was set to one.

workers and the unemployed move to high paying firms in booms a low paying ones in recessions) and procyclical match quality. Our results support the claims of Gertler and Trigari (2009) that cyclical up- and downgrading needs to be controlled for when testing for equal treatment. In that paper they also propose using MFE for this purpose.<sup>32</sup> Finally below where we allow for asymmetric responses of wages to output growth we find that the tests for excess new hire cyclicity remain wholly insignificant. The excess cyclicity found above is not therefore a consequence of estimating subject to a single elasticity. Summing up this section, our empirical results are broadly supportive of equal treatment and of the idea that the findings of unequal treatment in numerous other papers may be down to the failure to control for procyclical upgrading to better paying firms and procyclical upgrading in match quality.

## 4.2 Asymmetric Wage Adjustment

The purpose of this section is to establish a key stylized fact about the comovements of output and wages, namely that they positively comove when output is rising but are not well related when it is falling. As we have noted this property is a salient prediction of Snell and Thomas’s (2010) equal treatment wage contracting model as we will show later. Of course the definition of downswings and upswings is not unambiguous and given that the target is to see if the theoretical model can match the time series co-properties we identify here, we could choose any measure of the cycle. However output growth is not only a familiar and natural measure of up- and downswings (a recession in many countries is defined as two quarters of consecutive negative GDP growth) but is a natural metric suggested by the model; in the model productivity (growth) is the sole exogenous input and when productivity rises (falls) so does output.

We analyze asymmetric adjustment of wages to output using the three sets of wage year effects (for incumbent workers, ue new hires, and ee new hires) that we estimated above. Here we focus only on those year effects derived where we have controlled for MFE in (1). Figure 1 plots the first differences of the year effects (“wage growth”) for the three types of worker as a scatter plot

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<sup>32</sup>Gertler et al. (2016), who use the Survey of Income and Program Participation (SIPP) and also estimate semi-elasticities separately for new hires from unemployment and from job-to-job transitions, find that in a WFE specification the ue semi-elasticity is not significantly different from the incumbent one and the ee one is significantly larger in absolute size. This may be partly accounted for by the difference in labor markets between Germany and the US, and the much shorter period each worker is followed in the SIPP. This casts however some doubt on the generality of their hypothesis that it is job to job cyclical upgrading that is behind the increased cyclicity of new hires.

against the growth in output. We provide separate scatters for positive and negative output growth respectively. All graphs in Figure 1 employ the same scale.

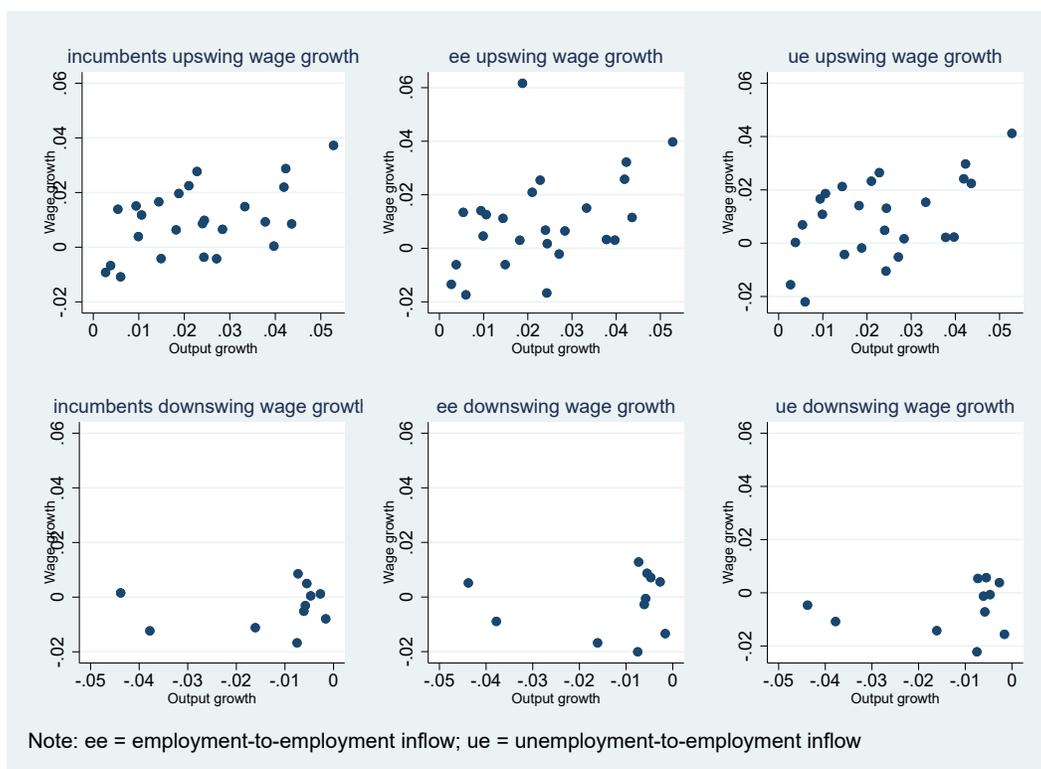


Figure 1: Scatter Plots of First Differences of the Year Effects (“Wage Growth”) for Three Types of Workers Against Output Growth.

We may make three heuristic observations. Firstly, all three types of workers have wage growth that display a “hockey stick” pattern; a clear positive slope during upswings but relatively flat in downswings. Second, there seems to be a markedly higher variance of wage growth in upswings than downswings (the ratio of upswing over downswing wage growth (%) variance for the three types of worker are 2.54, 2.55, and 2.61, respectively). This higher upswing wage growth variance is not coincident with higher upswing output variance; the latter is practically the same as its downswing counterpart. Thirdly, the upswing slopes appear to be similar in each of the three graphs. These three observations are what we would expect from a world of wage contracting subject to equal treatment. Our model (below) predicts that in most downswing periods, wages will most likely fall at an exogenously given rate — a rate below that (in absolute value) of the fall in productivity. This should generate downswing wages that are relatively less volatile and less well related to productivity growth compared with those in upswings; consistent with the first two observations.

The third observation is consistent with equal treatment — an assumption that underpins our model. We now re-examine these apparent data features more formally via a regression model.

The second-stage regression model we use to characterize the time series co-properties of wage growth for the three types of workers and output is

$$\Delta\beta_t^k = \alpha + \gamma_u^k \Delta y_t^+ + \gamma_d^k \Delta y_t^- + \varepsilon_t, \quad k = I, E, U \quad (4)$$

where  $\Delta y_t^+$  ( $\Delta y_t^-$ ) equals  $\Delta y_t$  if output is growing (contracting) and is zero otherwise.

Table 2 gives estimates of  $\gamma_u^k$  and  $\gamma_d^k$  for each of the three types of worker along with t-tests of their equality. The table also gives the estimates obtained when we replace the three sets of year effects in (1) with one single set of common year effects — effectively imposing the constraint that all workers have the same wage cyclicalities. These elasticities are denoted  $\gamma_u^A$  and  $\gamma_d^A$  (“A” for “All”).

As we have noted already, the model predicts that when output falls wages will likely fall but at a rate substantially lower than the output fall. By contrast in upswing periods, wages and output are likely to move closely together — particularly when we hit full employment.<sup>33</sup> These properties of the model are borne out in estimates of the  $\gamma$ 's in Table 2.<sup>34</sup> The downswing elasticities are close to zero and wholly insignificant whilst the upswing counterparts are positive and highly significant. Revisiting the equal treatment hypothesis, the final four columns give t-tests of equality of the incumbent up- and downswing elasticities with the corresponding ones for ue's and ee's respectively. We see that the differences between the estimates across the three worker types are small and wholly insignificant. Again this is consistent with the impressions gained from the scatter plots. The “A” column shows the up- and downswing elasticities when we impose them to be equal across worker types. They are of course close to their worker-type counterparts. It is these values that we will try to match in our simulations below.

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<sup>33</sup> Once at full employment and when productivity continues to grow, output and productivity move one for one. However our definition of upswings also include periods when we move from a recessionary (positive unemployment) state to a boom state (zero unemployment). In such periods wages rise less than productivity because their starting point is relatively high due to downward rigidity.

<sup>34</sup>We also included an up- and downswing dummy to allow for an “up/down” change in mean, but it was not significant and made only a marginal difference to the estimates.

Table 2: Upswing and Downswing Elasticities

$k$	$I$	$E$	$U$	$A$	$t_{\gamma_d^E - \gamma_d^I}$	$t_{\gamma_u^E - \gamma_u^I}$	$t_{\gamma_d^U - \gamma_d^I}$	$t_{\gamma_u^U - \gamma_u^I}$
$\gamma_d^k$	0.118 (0.158)	0.039 (0.182)	0.126 (0.128)	0.119 (0.160)	-1.26	0.69	0.09	1.31
$\gamma_u^k$	0.480 (0.122)	0.509 (0.143)	0.565 (0.138)	0.494 (0.124)				

Notes: Estimates of the parameters in equation (4) with robust standard errors in brackets. Columns  $I$ ,  $E$  and  $U$  contain estimates for incumbents, new hires from employment, and new hires from unemployment respectively. Column  $A$  (“All”) displays estimates for the case where a single set of year effects for all workers are extracted from the panel. The columns headed  $t_{a-b}$  display t-ratios for the test of  $a = b$ .

## 5 A Simple Equal Treatment Wage Contracting Model

In this section we analyze the extent to which a simple equal-treatment wage contracting model can reproduce the empirical characteristics of output, wages, and unemployment of the West German data. Whilst the main focus is on the ability to match the (asymmetric effect) estimates in Table 2 we also assess the model’s ability to match the univariate time series properties for each of the three variables and their sample covariances. We look at two versions of the model. The first is the model of Snell and Thomas (2010). We give a brief overview of the model below. We then consider an extension with habit formation. Like its predecessor this version is very parsimonious with a small number of parameters (four) and uses total factor productivity as its single exogenous driving process. We will find that it performs somewhat better in matching relationships in our data. Finally we also briefly look at a model with habit formation but *without* equal treatment in order to determine whether and how the different assumptions interact with each other.

### 5.1 The Model

#### Baseline Model (Snell and Thomas (2010))

Time runs  $t = 1, 2, \dots, T$ . There are a large (fixed) number of identical risk neutral firms producing a homogeneous good using only labor as an input in a diminishing returns (Cobb-Douglas with

fixed capital) production process where production is subject to a shock common across firms  $a_t$ :

$$F(l_t, a_t) = a_t l_t^\alpha, \quad (5)$$

where  $l_t$  is employment,  $\alpha \in (0, 1)$ , and  $a_t$  follows a finite state Markov process with transition probabilities  $\pi_{a,a'}$  and  $a_1$  given. There is a continuum of risk averse workers, of measure 1 relative to the the number of firms, with utility  $u(c) = c^{1-\gamma}/(1-\gamma)$ ,  $\gamma > 0$ ,  $\gamma \neq 1$ , where  $c$  is consumption. Workers choose to work full time or not to work at all, but cannot borrow or save so that consumption equals real wage  $w$  or unemployment benefit  $b$ . Labor markets are competitive and in equilibrium, all workers wish to work (no disutility of work). We assume that  $a_t > b$  so that the “spot wage” is  $a_t$  (equal to the marginal product of labor when employment is 1).<sup>35</sup>

At each date, a firm must pay its workers the same wage (equal treatment, no human capital).<sup>36</sup> At the start of each period  $a_t$  is revealed, before firms hire workers. A firm commits at the outset ( $t = 1$ ) to a long-term history-dependent wage contract  $(w_t(h_t))_{t=1}^T$  where  $h_t = (a_1, a_2, \dots, a_t)$ ; because of equal treatment this also fixes the wages offered to new hires since they simply slot into the existing wage contract. It is assumed that exogenous labor turnover  $1 - \delta \in (0, 1)$  is sufficiently high relative to shocks to guarantee that at least some new workers per firm are employed each period.<sup>37</sup> Firms and workers discount the future with respective factors  $\beta_f$  and  $\beta_w$ . We assume that  $(1 - \alpha)\gamma > 1$ .

The lifetime utility of an employed worker at the start of period  $t$  is denoted  $V_t$ , and that of an unattached worker  $\chi_t$ . The latter is also the utility an employed worker gets if she quits at the start of  $t$ . To be able to hire and to avoid losing its workforce firms must at least match  $\chi_t$  (there is no on-the-job search, although the equilibrium is robust to this). We have

$$V_t = u(w_t) + \beta_w E[\delta V_{t+1} + (1 - \delta)\chi_{t+1} \mid h_t], \quad (6)$$

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<sup>35</sup>The equilibrium wage contract and employment are independent of  $b$ , provided only that parameters are such that working at the putative equilibrium wage offers a higher utility than being unemployed. We assume below that this is satisfied, and for our calibrations it is satisfied for any reasonable replacement ratio (we do not need to calibrate  $b$  for the above reason).

<sup>36</sup>Snell and Thomas (2010) show that equal treatment is a property of optimal contracts under “at will” contracting. See Menzio and Moen (2010) for a similar argument with a frictional labor market.

<sup>37</sup>That is, we shall assume that optimal contracts involve hiring in every contingency and ignore the possibility of layoffs. Snell and Thomas (2010) derive sufficient conditions that guarantee this is optimal.

with  $V_{T+1}, \chi_{T+1} = 0$ , and we assume all unattached workers have an equal chance of finding work within the current period:

$$\chi_t = \frac{l_t^* - \delta l_{t-1}^*}{1 - \delta l_{t-1}^*} V_t^* + \frac{1 - l_t^*}{1 - \delta l_{t-1}^*} U_t, \quad (7)$$

where  $*$  denotes an equilibrium value,  $U_t$  is the discounted utility of a worker who fails to find work at  $t$  given by  $U_t(h_t) = u(b) + \beta_w E[\chi_{t+1} | h_t]$  and  $1 - \delta l_{t-1}^*$  is the measure of workers unattached at  $t$ . ( $\chi_1 = V_1^*$  as wages are flexible at  $t = 1$  and the initial period is assumed to be one of full employment.) The problem faced by the representative firm, which takes  $(\chi_t)_{t=1}^T$  as parametric, is then:

$$\max_{(w_t(h_t) \geq 0)_{t=1}^T, (l_t(h_t) \geq 0)_{t=1}^T} E \left[ \sum_{t=1}^T (\beta_f)^{t-1} (F(l_t(h_t), a_t) - l_t(h_t) w_t(h_t)) \right] \quad (\text{Problem A})$$

subject to the participation constraint for all positive probability  $h_t, T \geq t \geq 1$ ,

$$V_t(h_t) \geq \chi_t(h_t). \quad (8)$$

The firm has two conflicting objectives: a) to insure its risk-averse workforce by limiting the variability of wages over time and across states, and b) to take advantage of positive unemployment rates in bad times by lowering the wages of new hires. The requirement to pay new hires and incumbents the same results in a compromise where wages do fall in bad times but where the fall does not exceed some maximum rate. Consider the consequence of a negative productivity shock following a period when there was full employment. If the shock is sufficiently large, wages will not fall far enough to clear the labor market. At this point the firm gets no benefit from a further reduction in the wage because the extra risk-premium that has to be paid to earlier hires to compensate them for wage variability exactly equals the benefit from hiring more cheaply. Any extra slackness in the labor market leading to reductions in the utility needed to be offered to new hires, has no additional effect on the optimal wage contract. There will then be involuntary unemployment and this will persist until either productivity recovers or until wages have fallen sufficiently far to clear the labor market — whichever happens first. When productivity recovers and is rising so that there is full employment, however, the firm must allow wages to respond fully to it in order to be able to make new hires (to replace workers who are separated), and to prevent incumbents from quitting and immediately joining other firms (workers are fully mobile).

**Proposition 1** (i) *In an equilibrium with positive hiring, employment is at a level such that the marginal product of labor is equal to the contract wage,*

$$l_t = (w_t/\alpha a_t)^{-\frac{1}{1-\alpha}}; \quad (9)$$

(ii) *Define  $\hat{w}_{t+1}(w_t, a_t, a_{t+1})$  to be the solution to*

$$\left(\frac{w_{t+1}}{w_t}\right)^\gamma = \lambda \left(\frac{w_t a_{t+1}}{w_{t+1} a_t}\right)^{-\frac{1}{1-\alpha}}, \quad (10)$$

where  $\lambda \equiv \delta\beta_w/\beta_f$ . Then wages will satisfy  $w_{t+1} = \max\{\hat{w}_{t+1}, a_{t+1}\}$ ,  $t > 1$ , with  $w_0 = w_1 = a_1$ .<sup>38</sup>

The proposition allows for the dynamics of wages and employment to be computed in a very simple recursive manner, so that current wages and employment depend only on current and lagged productivity and lagged wages. Letting a  $\tilde{\cdot}$  above a variable denote a logarithm we can rewrite the wage dynamics as:

$$\begin{aligned} \tilde{w}_{t+1} &= \max\{(A_{t+1} + \tilde{w}_t), \tilde{a}_{t+1}\}, \\ \text{where } A_{t+1} &= \frac{1-\alpha}{(1-\alpha)\gamma-1} \tilde{\lambda} - \frac{1}{(1-\alpha)\gamma-1} \Delta \tilde{a}_{t+1} \end{aligned} \quad (11)$$

and

$$\tilde{l}_{t+1} = \frac{1}{1-\alpha} \log \alpha + \frac{1}{1-\alpha} \tilde{a}_{t+1} - \frac{1}{1-\alpha} \tilde{w}_{t+1}. \quad (12)$$

### Extension: External habits with equal treatment

We extend the baseline model to include external habits. As we will discuss later, this improves the fit somewhat and also leads to a more reasonable calibration.

The utility function is  $u(c, H) = (c - H)^{1-\gamma}/(1-\gamma)$ ,  $\gamma > 0$ ,  $\gamma \neq 1$ , where  $c$  is consumption

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<sup>38</sup>An outline proof for this and the other claims in this section are provided in Appendix C.

and  $H$  is an external habit. For  $H$  we follow standard practice (see, e.g., Dennis (2009)) and take  $H_t = \mu c_{t-1}$ , where  $c_{t-1}$  is lagged consumption of the relevant reference group and  $\mu \in [0, 1]$ ; for an employed worker we assume that the reference group is the continuously employed so that the habit, denoted by  $H_t^E$ , is  $\mu w_{t-1}$ , regardless of whether or not the worker is a new hire. For an unemployed person the reference group is the continuously unemployed so that the habit, denoted by  $H^U$ , is  $\mu b$ . We take the initial habit to be  $H_1 = \mu a_1$ .<sup>39</sup>

With this change, Proposition 1 still holds<sup>40</sup> but with  $\hat{w}_{t+1}(w_t, a_t, a_{t+1})$  replaced by  $\hat{w}_{t+1}(w_{t-1}, w_t, a_t, a_{t+1})$  defined as the solution for  $w_{t+1}$  to

$$\left(\frac{w_{t+1} - \mu w_t}{w_t - \mu w_{t-1}}\right)^\gamma = \lambda \left(\frac{w_t a_{t+1}}{w_{t+1} a_t}\right)^{-\frac{1}{1-\alpha}}.$$

### External habits with unequal treatment (UT)

We also benchmark the baseline model against a model with external habits but in which each cohort is treated independently. In this model the absence of the equal treatment friction implies there is no unemployment. The model also implies that each cohort of hires receive wages according to the state of current and prospective future productivity. These features are at odds with our empirical results above; however examining the properties of this model shines a light on the effectiveness of our equal treatment friction in explaining key features of employment wage and output data.

We maintain the assumption that firms can commit to contracts, and assume that workers can also commit.<sup>41</sup> There will be continuous full employment as firms can always reduce the wages of the cohort to be hired currently until the labor market clears. Commitment together with risk-neutrality of the firm implies that optimal contracts will stabilize the marginal utility of consumption (net of habit) for each cohort across time. As a result each cohort's wages vary with

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<sup>39</sup>As with  $b$ ,  $H^U$  does not impact on the solution provided employment is always preferable to unemployment, which can be seen to hold as long as negative shocks to  $a_t$  are not too large.

<sup>40</sup>See Section C.1 of the Appendix. Existence and uniqueness can be established as in Snell and Thomas (2010) *mutatis mutandis* under analogous conditions that guarantee positive hiring (negative shocks cannot be too large). The proof that a relaxed version of Problem A is a concave program extends to the model with habits.

<sup>41</sup>We assume a) that mobility costs are sufficiently high that incumbent workers would not prefer to leave for a better contract on offer to later cohorts, and also b) that  $b$  is sufficiently low that an unemployed worker would never choose to wait for a better labor market. In the equal treatment model neither assumption is needed provided only that  $a_t > b$  for all  $t$ .

the value of the external habit, and this in turn impacts on the evolution of average wages. Details are provided in the Appendix in Section C.2.

## 5.2 The Fit of the Model With the Data

As noted above we follow standard practice and calibrate the model to German data. We generate a synthetic productivity series with the same time series properties corresponding to the actual TFP for Germany and “feed” this series to the model to generate model simulated time series for output, unemployment and wages. We then characterize the time series properties (and co-properties) of these three variables and compare these characterizations to those obtained from their real counterparts: annual time series data on wages (wage year effects), unemployment, and output for Germany from 1978 to 2014. We derive simulated series for the above models assuming  $\beta_f = \beta_w$ . In the *baseline model* if we calibrate  $\lambda$  using the empirical separation rate we find that wages track productivity too closely. However, in an extension to the baseline model in Snell and Thomas (2010), it is shown that if workers have heterogeneous separation rates then  $1 - \lambda$  becomes the smallest separation rate of all workers at the representative firm.<sup>42</sup> This raises a problem as then  $\lambda$  has no empirical counterpart. In the *habits model*, on the other hand, we calibrate  $\lambda$  using our data on completed tenure. However the habits model also has an unidentified parameter,  $\mu$  that, as with the  $\lambda$  of the baseline, is not identified empirically. We resolve both of these issues by choosing  $\lambda$  in the baseline model and  $\mu$  in the habits model, respectively, to target the standard deviation of unemployment. For the *UT model* we maintain  $\mu$  at the level used in the habits model.

With respect to the other parameters, we use  $\alpha = 0.66$ ,  $\gamma = 4$ . In our model (and in growth/RBC models)  $\alpha$  is the labor share so 0.66 is a reasonable choice. In the macro literature values of  $\gamma$  anywhere between 1 and 10 are in common usage so 4 seems likewise reasonable. The value of baseline  $\lambda$  that allows us to hit the standard deviation of unemployment is 0.988. The value of  $\mu$  in the habits model that achieves the same end is 0.45. The average of completed tenure lengths in 2014 in our sample is around 7.5 years giving us a value for  $\lambda$  in the habits model of 0.87. Unlike Baseline and Habits the *UT model* requires a value for the common discount factor

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<sup>42</sup>That is, the equilibrium contract of the baseline model with homogeneous  $\lambda$ , as in Section 5.1, can be shown to be an equilibrium of the model with heterogeneous separation rates across workers where the minimum separation rate equals  $1 - \lambda$ . Intuitively the participation constraint of the type with the lowest separation constraint becomes the constraint that matters in this equilibrium. Alternatively, if  $\beta_w > \beta_f$ ,  $\lambda$  is greater than  $\delta$ .

$\beta_f = \beta_w$ . We set this to 0.97, a value appropriate for annual data although the results were largely insensitive to other values within a reasonable range.

Data for TFP<sup>43</sup> were taken from the Federal Reserve Bank of St Louis (FRED). This data shows (log of) TFP to be a unit root so we deal henceforth with productivity growth (the change in log TFP) which we denote as  $m_t$ . We summarize the time series properties of  $(m_t)$  using a simple AR(1) process:<sup>44</sup>  $m_t = \rho_p m_{t-1} + \varepsilon_t$ , with  $var(\varepsilon_t) = \sigma_p^2$  and where we take  $\varepsilon_t$  to be normally distributed.<sup>45</sup> The values for  $\sigma_p$  and  $\rho_p$  were chosen to match those obtained using the FRED time series from 1978 to 2014. The standard deviation was 1.7% and the autocorrelation coefficient was 0.36.

Both the Baseline and Habits versions of the model have three endogenous stationary variables; wage growth, output growth and unemployment (equivalently, employment, as labor supply is assumed constant) whereas the UT model has only output and wage growth (unemployment is zero). We simulate the models using the parameter configurations above and assess their ability to match the time series properties of the actual data including of course the key property of asymmetric cyclical response of wage growth to output growth. As we have a relatively small number of observations in the data (36 annual data points) we use a parsimonious AR(1) process to summarize the univariate dynamic time series properties of the three variates. Given that the model assumes equal treatment we use  $\Delta\beta_t$ , the “common” year effects from (4), as our wage growth measure. Results for the AR(1) coefficients ( $\rho_z$ ) and the standard deviation ( $\sigma_z$ ) of each data series and their model counterparts are given in the top half of Table 3. Further down the table we give the correlations between the variates together with the up- and downswing elasticities estimated above, again from the data and the two models.<sup>46</sup>

Given the parsimony of the first two models — the habits version has only four parameters and

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<sup>43</sup>Unlike all other data in the paper, TFP is only available for the whole of Germany after 1991. This is not a major issue; we only use the series indirectly to calibrate the time series properties of the synthetic simulated TFP series and we would not expect these properties to be much changed by the inclusion of the old East Germany after 1991.

<sup>44</sup>We add a 1% p.a. deterministic trend to match the trend found in the FRED data

<sup>45</sup>For the baseline model (but not for the habits model) the productivity draws had to be truncated where they fell outside the model’s permitted limits. The persistence and standard deviation of the TFP input were adjusted to make sure that ex post its AR(1) representation was as in the data but of course the truncation operation implies that the TFP input was not normally distributed. We revisit this issue below.

<sup>46</sup>Estimates derived from a single simulated time series of 1,000 periods. The productivity growth shock is normally distributed for the habits models and truncated normal for the baseline — see footnote 47 below.

Table 3: Data and Model Properties of Wage Growth ( $\Delta\beta_t$ ), Unemployment ( $u_t$ ) and Output Growth ( $\Delta y_t$ ).

<i>Data Variable(z)</i>	$\rho_z$	$\sigma_z$			
$\Delta\beta$	0.52	1.31			
$\Delta y$	0.27	2.20			
$u$	0.84	1.71			
<i>Baseline Model Variable(z)</i>					
$\Delta\beta$	0.39	1.61			
$\Delta y$	0.19	2.28			
$u$	0.61	—			
<i>Habits Model Variable(z)</i>					
$\Delta\beta$	0.44	1.64			
$\Delta y$	0.06	2.30			
$u$	0.45	—			
<i>UT Model</i>					
$\Delta\beta$	0.98	0.41			
$\Delta y$	0.36	1.70			
	$cor(\Delta\beta, \Delta y)$	$cor(\Delta\beta, u)$	$cor(\Delta y, u)$	$\gamma_u^A$	$\gamma_d^A$
<i>Data</i>	0.61	-0.24	-0.05	0.50	0.12
<i>Baseline Model</i>	0.75	-0.42	-0.59	0.60	0.25
<i>Habits Model</i>	0.64	-0.26	-0.59	0.74	0.23
<i>UT Model</i>	0.45			0.13	0.10

Notes: Estimates of the AR(1) coefficients ( $\rho_z$ ) and standard deviation ( $\sigma_z$ ) of wage growth (%), output growth (%), and the unemployment rate (%), are from the data and models (top half of table). The bottom half of the table gives correlation coefficients between the three variates ( $cor(a, b)$ ) along with the upswing and downswing elasticities from the data and the models. The standard deviation of unemployment is the target for the baseline and habits models - hence the void entry in the relevant cells. In the unequal treatment (UT) model unemployment is zero and output equals productivity.

the baseline three — they reproduce the main features of the data quite well. All quantities have the correct sign. The upswing elasticity found in the models is similar to that found in the data. As explained above in footnote 33 the models predict that this quantity will be less than one and this is borne out by the data. In both of the models and in the data the downswing elasticities are small. The standard deviation of output and wage growth generated by both models is fairly close to that found in the data and the correlation coefficients from both models — aside from that between unemployment and output growth — are a reasonable fit with the data. Both models underestimate the persistence of unemployment somewhat.

Comparing the performance of the baseline with its habit counterpart we see neither is uniformly

superior. The habits model underestimates unemployment persistence more than the baseline but does better on wage growth persistence. An important reason to prefer the habits version over the baseline is that the latter places more severe restrictions on the maximum size of downward productivity shock that is admissible. In fact the two largest TFP falls we observe in the data are not admitted by the baseline under its current calibration.<sup>47</sup> The restriction on the support of productivity shocks may be eased by lowering the baseline model’s value of  $\lambda$  but doing so undermines its fit with the data. We therefore consider the habits version to be more general in terms of the economies it may represent.

With respect to the *UT* model we see the properties of aggregate wage growth are markedly at odds with the data. Wage growth has far too much persistence<sup>48</sup> and only one third of the volatility of the data counterparts. The asymmetry of wage growth over the cycle has disappeared — unsurprisingly as in the *UT* model the marginal utility of each cohort is kept constant regardless of the state of the cycle and new hire wages respond symmetrically to upturns and downturns. Interestingly it is not the existence of habits that is responsible for these properties but the absence of equal treatment; removing habits (setting  $\mu = 0$ ) changes the *UT* results little. In particular, the extreme persistence of aggregate wage growth in the *UT* model derives not from habits but from the fact that under unequal treatment and perfect insurance, new hire wages for each joining cohort will be attuned to current and prospective future productivity conditions at the time of hiring and, thanks to insurance, will only move during the worker’s tenure to match the external habit. Consequently, the growth in the aggregate wage — which is an infinite weighted moving average of current and past new hire wage growth will be extremely persistent.<sup>49</sup> This is in stark contrast to the two equal treatment models and is seen most clearly in upturns in these models when there is no amplification of the persistence of this process because all wages respond fully to

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<sup>47</sup>As we noted in footnote 45, in the baseline simulations we have to truncate productivity draws that are too large by resetting them to the maximum allowed fall. The variance and persistence of the TFP input was adjusted to ensure that after such truncation the AR(1) parameter was 0.36 and unconditional standard deviation was 1.6% as per the data.

<sup>48</sup>The AR(1) root in aggregate wage growth displayed in the table is very close to one which hints at a possible unit root. However in the appendix we identify a stable solution to the AR(3) form for wages implied by the model, which implies that if productivity (growth) is stationary wage growth will be too.

<sup>49</sup>To take a concrete example suppose there is unequal treatment but no habits; then the solution for the new hire wage is the expected net present value of current and future productivity. Now suppose that productivity  $a_t$  follows a simple random walk with white noise innovation  $\varepsilon_t$ . Then  $w_t^t = a_t$ . The aggregate (average) wage at  $t$  is equal to  $(1 - \delta) \sum_{i=0}^{\infty} \delta^i a_{t-i}$  and aggregate wage growth will be  $(1 - \delta) \sum \delta^i \varepsilon_{t-i}$ . In this case the persistence of aggregate wages — as measured by its AR(1) coefficient — is  $\delta$  which in our calibration is 0.87. If we add some persistence to epsilon then this value will be even higher. Note that this persistence exists in spite of the fact that the new hire wage change is white noise. A similar argument applies to the low s.d. we find in wage changes.

current TFP changes. In sum, then, the success of the (equal treatment) habits model in matching the data is down to the interaction of the equal treatment assumption and habits rather than habits alone. The interaction arises due to the higher elasticity of marginal utility with respect to consumption when there are habits: this means that wage falls in recessions are smaller to avoid excessive marginal utility variation. In the  $UT$  model the insurance motive applies separately to each cohort but, critically, does not link wages of different cohorts.

We conclude that despite their simplicity, both the baseline model with heterogeneous quit rates and the habits model with constant quit rates, can account for most of the features and co-features of wages, unemployment, and output, especially the asymmetrical response of wages to output. The existence of the equal treatment friction is essential to the success of both of these models.

## 6 Caveats, Robustness Checks and Methodological Issues

### *Issues with the early years and reunification*

Here we examine two issues that may impact our estimates and for which we need a robustness check. First of all we had to drop workers who were incumbents before January 2<sup>nd</sup>, 1975 because their tenure is censored. This gives us relatively few incumbents in the early years of the sample. Consequently, the precision of the year effects in these early years will be relatively low and this in turn may reduce the power of any test of extra new hire cyclicity. Omitting these early years and focusing the test on the relatively more precise new hire and incumbent year effects may improve the test power. By 1982 we get a ratio of new hires to incumbents that roughly represents that in the uncensored years of the sample (see Table A1) and so we re-estimate (3) and (2) from 1982–2014 dropping the early years. The t-tests for equal treatment —  $t_{\gamma E - \gamma I}$  and  $t_{\gamma U - \gamma I}$  — in Table 1 are now  $-.92$  and  $-.46$  respectively.

Second, reunification and the years just after were traumatic years for Germany. We do not have a view on how this may impact labor markets in general and the asymmetrical response of wages to output in particular. Nonetheless we assess the impact on the main results of dropping the years 1989 to 1992 in the second stage regression. We re-estimated upswing and downswing elasticities for the “*All*” case above (the case where we estimate a single wage year effect for incumbents and

new hires together). The numbers are virtually unchanged at 0.470 and 0.110 respectively, as are their standard errors.

### *The Issue of Small Samples*

An issue with our study, and indeed with all work of this nature on wage cyclicality, is the small number of observations we have to work with — 36 years in all. Like the rest of the literature we only have the business cycles that an economy undergoes and these are small in number. However we can re-estimate the “*All*” up and downswing elasticities for each of six broad industrial sectors. While this offers up no extra business cycles it allows us to see whether the asymmetry in response to the business cycle applies at a more disaggregated level (and may allay fears that our results hinge on just 36 data points).

Table 4: Upswing and Downswing Elasticities ( $\gamma_u^A$  and  $\gamma_d^A$ ) for six Broad Industrial Sectors ( $S1, \dots, S6$ )

	<i>All</i>	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>	<i>S5</i>	<i>S6</i>
$\gamma_d^A$	0.110 (0.164)	0.087 (0.313)	0.528 (0.096)	-0.350 (0.291)	-0.132 (0.155)	0.016 (0.130)	-0.106 (0.209)
$\gamma_u^A$	0.470 (0.136)	0.448 (0.116)	0.545 (0.112)	0.396 (0.150)	0.698 (0.208)	0.482 (0.108)	0.431 (0.139)

Note: Robust standard errors in brackets. The six sectors are: (S1) agriculture, hunting and forestry (A), fishing (B), mining and quarrying (C), (S2) manufacturing (D), (S3) electricity, gas and water supply (E), (S4) construction (F), (S5) wholesale and retail trade; repair of motor vehicles, motorcycles and personal household goods (G), and (S6) hotels and restaurants (H), transport, storage and communication (I), financial intermediation (J), real estate, renting and business activities (K), public administration and defense; compulsory social security (L), education (M), health and social work (N), other community, social and personal service activities (O), private households with employed persons (P), and extra-territorial organizations and bodies (Q).

The up- and downswing  $\gamma$ 's are displayed in Table 4 under the columns headed  $S1, S2, \dots, S6$ . Apart from sector S2 (manufacturing) where the downswing elasticity is significant and only marginally less than its upswing counterpart the main result remains intact: downswing elasticities are wholly insignificant and either perverse in sign or close to zero whilst their upswing counterparts are relatively significantly positive.

### *Cyclical Variation in Hours Worked*

One drawback of the BeH noted above is that it documents total earnings rather than earnings per hour. It was for this reason that we chose to work with only full-time employees because nearly all such workers work a standard length of week in Germany. While it is not possible to show definitively the hours worked by the workers in our panel are unrelated to the output growth, there is some extraneous evidence that cyclical of hours is very low as we claim.

We have obtained estimates of the average number of hours worked in a year for West German full-time workers for each of the years 1977 to 2009.<sup>50</sup> These data are confidential but we are able to summarize their time series properties. We measure the log of hours which we henceforth refer to as just “hours”. We find that hours<sup>51</sup> are nonstationary (*ADF* test with quadratic trend  $-0.73$  compared with a 5% critical value of  $-3.8$ ) and so we first difference them to analyze their covariation with output growth.

The growth in hours is virtually acyclical; regressing it on output growth gives a coefficient of 0.07 with a (robust) t-ratio of 0.80. The corresponding estimate for HP detrended (log) hours on HP detrended output is 0.10 with a (robust) t-ratio of 1.36 respectively.

Additionally the volatility (standard deviation) of hours growth is only one quarter that of output growth whilst the volatility of HP filtered hours is only one-fifth that of HP filtered output. Overall, although there appears to be some variation in hours worked by full time workers in Germany, such variation would appear to be small and acyclical.

Finally, we note that even if hours were procyclical this would lead to us overestimate the extent of wage cyclical and yet we find cyclical in downswings that is close to zero and insignificant. Similarly, with respect to equal treatment, it is hard to see how procyclical in hours for all workers, per se, could lead to false inferences on this matter.<sup>52</sup>

*Changing the Definition of Output* In all of the above analysis we have used real (West German) GDP per capita as our measure of output to try and isolate the impact (if any) of the secular exogenous rise in the size of the West German economy. It would be prudent to assess the impact

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<sup>50</sup>Source: IAB Arbeitszeitrechnung, data status March 2013.

<sup>51</sup>Once again we allow for a quadratic trend — weekly hours have a secular downward trend in Germany.

<sup>52</sup>In fact we would require *differential* procyclical of hours for this to happen. To see this suppose that an upswing (downswing) increases (decreases) new hire wages above (below) that of incumbents. For this to be hidden by the BeH it would be necessary for new hires to work fewer (more) hours than incumbents in the upswing (downswing).

of this normalisation by recomputing the main results for plain vanilla real GDP. These results – tabulated in the appendix – are numerically very close to those obtained above.

#### *Varying the definition of a newly hired worker*

So far we have defined a new hire spell as one where the worker has joined the firm for the first time. By definition of a spell and our selection criteria, this means a continuous period of employment within a calendar year with the spell end technically defined as December 31<sup>st</sup>. It would be interesting to see how the equal treatment test is affected by varying this definition to allow workers to remain new hires for a longer period of time — up to two years. In a sensitivity analysis we identify a “new hire” spell as above (as the first spell with a firm) but in addition we identify any second spell with the firm as a “new hire” too, when tenure at the end of the second spell is below 2 years ( $\leq 732$  days). In that case we also apply the information concerning the origin of the flow (from unemployment/nonemployment or employment) from the first to the second spell. Re-estimating unemployment semi elasticities ( $\gamma$ ) under this new definition of incumbents/new hires barely changes the results at all. In particular the tests for extra new hire cyclicality yield t-ratios of 0.17 and 0.71 for ee’s and ue’s respectively. The t-test for extra ue cyclicality with respect to output becomes  $-1.34$  whilst for ee’s the elasticity is marginally lower than that for incumbents.

#### *Collective bargaining*

Centralized wage bargaining in Germany occurs mostly at the sectoral level between unions and employers’ associations (Fitzenberger et al., 2013). In the late 1990s, approximately 73% of the workforce was covered by union agreements (Dustmann and Schönberg, 2009), although this has declined since. However there is reason to believe that the impact of these agreements on the cyclicality of wages is not substantial. First, clauses in collective contracts allow firms to deviate from sectoral agreements if jobs are at risk (Addison et al., 2007). Second, nearly 50% of firms that recognize a union pay wages above union wages (Dustmann and Schönberg, 2009).<sup>53</sup> These so-called wage cushions can easily be reduced if the economic situation worsens (Jung and Schnabel, 2011). According to Gerlach and Meyer (2010) “for the period before the reform of the Works Constitution Act (2002) most studies suggest that collective agreements and unions have only small, even zero effects on wages”. As discussed in Section 2 however, Gartner et al. (2013) find some differential

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<sup>53</sup>See also footnote 10.

in wage cyclicalities between workers covered by unions and those not covered.

## 7 Conclusions

As far as we know, this is the first paper not only to look at the empirical downward real rigidity of wages for workers including new hires, but also to simulate a microfounded model that is capable of matching observed regularities (Snell and Thomas (2010) only perform unemployment simulations).

We use a sample of West German workers drawn from a large administrative German panel dataset for the years 1978–2014 to estimate the elasticity of wages with respect to output. We define up- and downswings in terms of positive and negative output growth to allow asymmetrical response of wage growth to output growth. The key empirical feature to emerge from our study is that the elasticity of wage growth with respect to output growth is close to zero and insignificant for both new hires and incumbents in downswings. However in upswings the elasticity is around one half and highly significant for both classes of worker. We also find no extra cyclicalities in new hire wages when we use unemployment as the cyclical indicator instead of output. These results for new hires are particularly important: if wages are downwardly rigid in downswings then it suggests such workers cannot price themselves into jobs by “undercutting” incumbents and labor markets will not clear (see, e.g., Hall, 2005; Gertler and Trigari, 2009; Snell and Thomas, 2010). The paper also presented results that offered tentative support for equal treatment of newly hired and incumbent workers in both up- and downswings.

We further show that both a three and a four parameter version of Snell and Thomas’ (2010) equal-treatment contracting model can generate the asymmetrical response of wage growth to output growth seen in the data. The model(s) can also match moderately well the key dynamic time series features and co features of output and wage growth although both underestimate the persistence of unemployment and its correlation with output somewhat. It may be that adding search frictions to the model may improve its shortcomings and research along these lines is underway.

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## A Further Tables

Table A.1: No. of Spells of Incumbent and Newly Hired Workers

Year	Newly Hired	Incumbents
1978	536,480	860,131
1979	580,482	1,070,423
1980	562,231	1,254,231
1981	472,966	1,423,195
1982	383,748	1,535,036
1983	384,038	1,607,852
1984	421,761	1,650,744
1985	433,296	1,703,623
1986	480,197	1,829,471
1987	467,208	1,925,379
1988	501,192	2,008,610
1989	580,223	2,080,315
1990	674,453	2,164,259
1991	651,557	2,284,766
1992	569,494	2,394,251
1993	482,607	2,431,712
1994	496,822	2,428,188
1995	516,571	2,416,687
1996	481,872	2,408,716
1997	481,019	2,405,614
1998	524,318	2,392,430
1999	580,765	2,385,722
2000	601,915	2,445,300
2001	558,655	2,454,149
2002	471,745	2,444,711
2003	424,415	2,505,278
2004	395,014	2,473,805
2005	391,361	2,443,718
2006	441,206	2,449,759
2007	487,477	2,465,401
2008	474,157	2,506,474
2009	400,230	2,502,328
2010	462,299	2,502,616
2011	444,522	2,409,295
2012	430,893	2,480,722
2013	418,203	2,519,325
2014	432,368	2,521,718
Total	18,097,160	79,785,954

Note: Newly hired workers identified using the first employment spell in a firm.

Table A.2: GDP, CPI, Population, and Unemployment Rate

Year	Nominal GDP (in Mill. Euros)	CPI	Population (in 1,000)	Unemployment rate (in %)	Real GDP/Capita Change	Real GDP Change
1978	678,940	47.6	61322	4.3	0.0389	0.0384
1979	737,370	49.5	61439	3.8	0.0419	0.0439
1980	788,520	52.2	61658	3.8	0.0106	0.0142
<b>1981</b>	825,790	55.5	61713	5.5	-0.0161	-0.0152
<b>1982</b>	860,210	58.4	61546	7.5	-0.0075	-0.0102
1983	898,270	60.3	61307	9.1	0.0149	0.0110
1984	942,000	61.8	61049	9.1	0.0284	0.0240
1985	984,410	63.0	61020	9.3	0.0244	0.0239
1986	1,037,130	63.0	61140	9.0	0.0528	0.0549
1987	1,065,130	63.1	61238	8.9	0.0228	0.0244
1988	1,123,290	63.9	61715	8.7	0.0333	0.0413
1989	1,200,660	65.7	62679	7.9	0.0240	0.0400
1990	1,306,680	67.5	63726	7.2	0.0423	0.0597
1991	1,404,585	70.2	64485	6.2	0.0210	0.0331
<b>1992</b>	1,485,759	73.8	65289	6.4	-0.0055	0.0069
<b>1993</b>	1,503,858	77.1	65740	8.0	-0.0378	-0.0312
1994	1,556,575	79.1	66007	9.0	0.0038	0.0079
1995	1,606,164	80.5	66342	9.1	0.0094	0.0145
<b>1996</b>	1,625,847	81.6	66583	9.9	-0.0058	-0.0022
1997	1,664,512	83.2	66688	10.8	0.0027	0.0043
1998	1,711,722	84.0	66747	10.3	0.0182	0.0191
1999	1,751,665	84.5	66946	9.6	0.0144	0.0174
2000	1,799,706	85.7	67140	8.4	0.0099	0.0128
2001	1,856,557	87.4	65323	8.0	0.0397	0.0115
<b>2002</b>	1,879,896	88.6	65527	8.5	-0.0047	-0.0016
<b>2003</b>	1,888,205	89.6	65619	9.3	-0.0073	-0.0059
2004	1,933,051	91.0	65680	9.4	0.0060	0.0070
<b>2005</b>	1,960,396	92.5	65698	11.0	-0.0016	-0.0013
2006	2,038,803	93.9	65667	10.2	0.0243	0.0238
2007	2,142,032	96.1	65664	8.3	0.0271	0.0270
<b>2008</b>	2,180,829	98.6	65541	7.2	-0.0061	-0.0080
<b>2009</b>	2,088,073	98.9	65422	7.8	-0.0438	-0.0455
2010	2,191,138	100.0	65426	7.4	0.0378	0.0379
2011	2,298,449	102.1	64429	6.7	0.0436	0.0276
<b>2012</b>	2,345,295	104.1	64619	6.6	-0.0027	0.0003
2013	2,401,853	105.7	64848	6.7	0.0054	0.0089
2014	2,483,514	106.7	65223	6.7	0.0188	0.0247

Note: Identified downswing years for main results are indicated in bold year numbers. In the robustness check where we use real GDP, the year 2012 is not identified as a downswing year. Sources for the nominal GDP for West Germany: German Federal Statistical Office & the Federal Statistical Offices of the Federal States. Source German CPI: Federal Reserve Bank of St. Louis (FRED Economic Data). Source West German Population: German Federal Statistical Office. Source West German unemployment rate (in % of total civilian workforce): Sachverständigenrat.

## B Robustness Check — Using Real GDP

The following three tables show results of our robustness checks, using real GDP instead of real GDP per capita. Table B.3 shows results comparable to Table 1, Table B.4 comparable to Table 2, and Table B.5 comparable to Table 3.

Table B.3: The Cyclicalities of Incumbent and New Hire Wages

Wage Elasticity (Output)	$\gamma_{\Delta y}^I$	$\gamma_{\Delta y}^E$	$\gamma_{\Delta y}^U$	$t_{\gamma_{\Delta y}^E - \gamma_{\Delta y}^I}$	$t_{\gamma_{\Delta y}^U - \gamma_{\Delta y}^I}$
<i>MFE</i>	0.409 (0.091)	0.414 (0.115)	0.452 (0.094)	0.13	1.33
<i>WFE</i>	0.429 (0.102)	0.592 (0.122)	0.632 (0.150)	3.24	2.47

Notes: Estimates of the parameters in equations (2) using real GDP instead of real GDP/capita. Robust standard errors using equation in brackets.  $t_{a-b}$  is the t-test for  $a = b$ . MFE = worker-firm (match) fixed effects, WFE = worker fixed effects, superscript I = incumbent workers, and superscript E (U) = new hires from employment (unemployment).

Table B.4: Upswing and Downswing Elasticities

$k$	<i>I</i>	<i>E</i>	<i>U</i>	<i>A</i>	$t_{\gamma_d^E - \gamma_d^I}$	$t_{\gamma_u^E - \gamma_u^I}$	$t_{\gamma_d^U - \gamma_d^I}$	$t_{\gamma_u^U - \gamma_u^I}$
$\gamma_d^k$	0.055 (0.153)	-0.011 (0.176)	0.109 (0.128)	0.060 (0.158)	-1.22	0.76	0.71	0.80
$\gamma_u^k$	0.564 (0.080)	0.600 (0.112)	0.601 (0.109)	0.575 (0.083)				

Notes: Estimates of the parameters in equation (4) using real GDP instead of real GDP/capita with robust standard errors in brackets. Columns *I*, *E* and *U* contain estimates for incumbents, new hires from employment, and new hires from unemployment respectively. Column *A* (“All”) displays estimates for the case where a single set of year effects for all workers are extracted from the panel. The columns headed  $t_{a-b}$  display t-ratios for the test of  $a = b$ .

Table B.5: Data Properties of Wage Growth ( $\Delta\beta_t$ ), Unemployment ( $u_t$ ) and Output Growth ( $\Delta y_t$ ) using real GDP.

<i>Data Variable</i> ( $z$ )	$\rho_z$	$\sigma_z$			
$\Delta\beta$	0.52	1.31			
$\Delta y$	0.36	2.21			
$u$	0.84	1.86			
	$cor(\Delta\beta, \Delta y)$	$cor(\Delta\beta, u)$	$cor(\Delta y, u)$	$\gamma_u^A$	$\gamma_d^A$
<i>Data</i>	0.71	-0.24	-0.09	0.58	0.06

Notes: Estimates of the AR(1) coefficients ( $\rho_z$ ) and standard deviation ( $\sigma_z$ ) of wage growth (%), output growth (%) —using real GDP instead of real GDP/capita, and the unemployment rate (%), are from the data (top half of table). The bottom half of the table gives correlation coefficients between the three variates ( $cor(a, b)$ ) along with the upswing and downswing elasticities from the data.

## C Details for Section 5.1

### C.1 Proof of Proposition 1

**Proof.** We address the general equal treatment case with habits; the proof of Proposition 1 then follows from setting  $\mu = 0$ .

(i) This follows because with equal treatment and positive hiring in all states next period, the user cost of labor (see Kudlyak (2014)) equals the current wage (an extra hire at  $t$  can be offset by reduced hiring at  $t + 1$ ). (ii) Start from an optimal contract and take some  $h_t$  given. Increase  $w_{t+1}(h_t, a)$  by a small amount  $\Delta_{t+1} > 0$ , and change  $w_t(h_t)$  by  $\Delta_t < 0$  to leave an incumbent worker in the firm at  $t$  indifferent (otherwise leave the contract unchanged) so that:

$$\frac{\partial u(w_t, H_t^E)}{\partial w} \Delta_t + \pi_{a_t a} \delta \beta_w \frac{\partial u(w_{t+1}, H_{t+1}^E)}{\partial w} \Delta_{t+1} \simeq 0, \quad (13)$$

where we suppress the state arguments for presentational simplicity.  $V_\tau$ ,  $\tau \geq 0$ , is unchanged except at  $t + 1$  after  $(h_t, a)$  where it increases, so no participation constraints are violated. The change in discounted profits at  $h_t$  is  $\Delta P = -(l_t \Delta_t + \pi_{a_t a} \beta_f l_{t+1} \Delta_{t+1})$  (where  $l_{t+1} \equiv l_{t+1}(h_t, a)$  etc.). Using (13) to eliminate  $\Delta_t$  gives the change in profits as

$$\Delta P \simeq -\pi_{a_t a} \beta_f l_{t+1} \Delta_{t+1} + \pi_{a_t a} \delta \beta_w \frac{\partial u(w_{t+1}, H_{t+1}^E)}{\partial w} l_t \Delta_{t+1} \left( \frac{\partial u(w_t, H_t^E)}{\partial w} \right)^{-1}. \quad (14)$$

By optimality,  $\Delta P \leq 0$ , so rearranging and letting  $\Delta_{t+1} \rightarrow 0$  to make the approximation precise, we get:

$$\frac{w_{t+1} - H_{t+1}^E}{w_t - H_t^E} \geq \left( \frac{\lambda_t}{l_{t+1}} \right)^{\frac{1}{\gamma}}. \quad (15)$$

If the participation constraint at  $t + 1$  in state  $a$  is slack, then the above argument can be repeated but now front-loading wages ( $\Delta_{t+1} < 0$ ) to demonstrate the reverse inequality:  $V_{t+1}$  falls but by assumption the participation constraint (8) is not violated for small enough  $\Delta_{t+1}$ . In this case (15) holds with equality, and substituting for  $l$  from (9) and using the assumption that  $H_{t+1}^E = \mu w_t$  we get (10). Suppose that  $\hat{w}_{t+1}$ , is below the spot wage  $a_{t+1}$ . By definition of the latter, if the equilibrium wage was at  $\hat{w}_{t+1}$ , there would be excess demand for labor and a firm would improve its profits by setting  $w_{t+1} > \hat{w}_{t+1}$  (by a small amount) so being able to hire as many workers as it desires. Since all firms will want to do this, equilibrium can only be restored at  $w_{t+1} = a_{t+1}$ . At this point (8) will bind since a worker faces no unemployment risk and can move to another firm offering the same utility. A higher wage as a deviation by a single firm would imply (8) slack, and so violate (10). Equally the equilibrium wage cannot be above  $a_{t+1}$  as again (8) would be slack due to the existence of unemployment. The result follows. ■

## C.2 The Unequal Treatment (UT) Model

We consider the habit model without equal treatment, but with commitment by both workers and firms. (Firms cannot dismiss “expensive” workers; workers cannot move to higher paying jobs, due to, e.g., high mobility costs.) For simplicity we assume that there is symmetric discounting,  $T = \infty$  and we further suppose that agents have perfect foresight.<sup>54</sup> Each cohort is treated independently, and let  $\{w_t^\tau\}_{t=\tau}^\infty$   $t \geq \tau$  be the contract offered to the cohort hired at time  $\tau$ . As there is no

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<sup>54</sup>Recall that the equal treatment characterization applies to deterministic as well as stochastic productivity sequences, and so we can compare the models under the perfect foresight assumption; the persistence of wage changes in the UT model may be larger than if the evolution of future changes to TFP are unknown, but the argument of Footnote 49 concerning persistence amplification under UT applies to both deterministic and stochastic environments.

equal treatment friction, unemployment will be zero. By standard arguments the optimal contract between a worker and a firm will imply that the marginal utility of consumption is constant over time:

$$\frac{\partial u(w_t^\tau, H_t^E)}{\partial w} = \frac{\partial u(w_{t'}^\tau, H_{t'}^E)}{\partial w},$$

all  $t, t' \geq \tau$ , so consumption net of habit

$$w_t^\tau - H_t^E =: \kappa_\tau \tag{16}$$

is constant.  $H_t^E$  is now  $\mu$  times the average of wages over *all* cohorts employed at  $t - 1$  which we denote as  $\bar{w}_{t-1}$ :

$$H_t^E = \mu \bar{w}_{t-1} = \mu \left( (1 - \delta) w_{t-1}^{t-1} + \delta (1 - \delta) w_{t-1}^{t-2} + \delta^2 (1 - \delta) w_{t-1}^{t-3} + \dots \right) \tag{17}$$

given that full-employment turnover implies hiring is  $(1 - \delta)$  in each period. The equilibrium marginal product of labor (i.e., at  $l = 1$ ) is  $\partial F(1, a_t) / \partial l = a_t$ . Labor market equilibrium then requires

$$a_t + \beta \delta (w_{t+1}^{t+1} - w_{t+1}^t) + \beta^2 \delta^2 (w_{t+2}^{t+1} - w_{t+2}^t) + \dots = w_t^t, \tag{18}$$

where the terms involving wage differences capture the “capital gains” from an extra hire today relative to postponing the hire until tomorrow (or subtracting these from  $w_t^t$  would yield the “user cost” of labor (Kudlyak (2014))).<sup>55</sup> Using (16) we can rewrite (18) as

$$a_t = w_t^t + \beta \delta (\kappa_t - \kappa_{t+1}) (1 - \beta \delta)^{-1}, \tag{19}$$

and (17) as

$$H_t^E = \mu \left( (1 - \delta) w_{t-1}^{t-1} + \delta (H_{t-1}^E + (1 - \delta) (\kappa_{t-2} + \delta \kappa_{t-3} + \dots)) \right) \tag{20}$$

$$= \mu \left( H_{t-1}^E + (1 - \delta) (\kappa_{t-1} + \delta \kappa_{t-2} + \delta^2 \kappa_{t-3} + \dots) \right). \tag{21}$$

---

<sup>55</sup>We are assuming throughout that  $\{a_t\}$  is such that  $a_t > b$  and it never pays an unemployed worker to delay getting a job.

Lagging (21) one period and substituting back into (20), we get

$$H_t^E = \mu(1 - \delta)w_{t-1}^{t-1} + \delta\mu(H_{t-1}^E - H_{t-2}^E) + \delta H_{t-1}^E. \quad (22)$$

(19) and (22) together with the definition of  $\kappa_t$  constitute a third-order system with one unstable root, that can be written in terms of the average wage:

$$\bar{w}_t = \left(\frac{1}{\beta\delta} + \delta + \mu\right)\bar{w}_{t-1} - \left(\frac{\delta\mu + \delta}{\beta\delta} + \mu\right)\bar{w}_{t-2} + \frac{\mu\bar{w}_{t-3}}{\beta} - \frac{(1 - \delta)(1 - \beta\delta)a_{t-1}}{\beta\delta}$$

and that can be solved for the path of wages consistent with a transversality condition.